

Markscheme

November 2017

Mathematics

On-screen examination



This markscheme is **confidential** and for the exclusive use of examiners in this examination session.

It is the property of the International Baccalaureate and must **not** be reproduced or distributed to any other person without the authorization of the IB Global Centre, Cardiff.

The following are the annotations available to use when marking responses.

Annotation	Explication	Annotation	Explication	Shortcut
?	Unclear	AO	Award 0 marks	Alt+0
SC	Special case		Award 1 mark	Alt+1
MR	Misread	✓ 2	Award 2 marks	Alt+2
NWS	No working shown	✓ 3	Award 3 marks	Alt+3
ECF	Error carried forward	✓ 4	Award 4 marks	Alt+4
WITE	Words to that effect	✓ 5	Award 5 marks	Alt+5
BOD	Benefit of the doubt	✓ 6	Award 6 marks	Alt+6
AG	Answer Given	₹ 7	Award 7 marks	Alt+7
×	Highlight tool	₹ 8	Award 8 marks	Alt+8
0	Ellipse tool	₹ 9	Award 9 marks	Alt+9
T	On page comment tool	₹10	Award 10 marks	
SEEN	Seen	✓11	Award 11 marks	
λ	Caret - Omission	~ 12	Award 12 marks	
~~~	Wavy underline tool			

## The markscheme may make use of the following abbreviations:

## RM Assessor has the following annotations that should be used to award marks:

A0 only use to award a zero mark for an answer that has no merit eg, awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with

Marks awarded by stamping the tick

SEEN

Seen; must be stamped on all blank response areas and on concatenated responses



ECF Marks that can be awarded as error carried forward from previous results in the question

**BOD** Benefit of the doubt

MR misread

**NWS** no working shown

SC special case

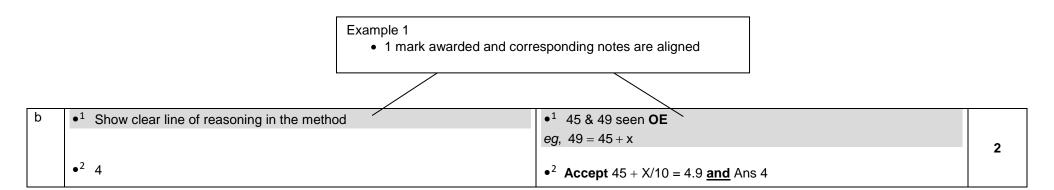
**OE** or equivalent

WTTE or words to that effect

AG Answer given

## The markscheme makes use of the following abbreviations:

• Bullet notation means award 1 mark – see example below



#### **Error Carried Forward (ECF) marks**

Errors made at any step of a solution affect all working that follows. In general, Error Carried Forward (ECF) marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or sinx > 1) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award ECF marks for a question part, there must be working present for that part.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (MR) is an error. ECF is normally awarded.

#### **General points**

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x² unless noted otherwise in the MS.
- b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e.** if the further working contradicts the correct answer, then the last mark cannot be awarded.
- c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or equivalent) e.g.  $\frac{1}{2}$  **OR** 1/2 **OR** 1÷2 and  $\frac{x}{2}$  x/2 **OR** x÷2
- e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- f) Special case marks (SC) can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- g) Accept seeing equation not in-line.
- h) Accept notation errors in intermediate steps.
- i) When a calculator screenshot is taken, accept not seeing the whole operation.
- j) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.

Questio	n Answers	Notes	Total
1 a	•¹ two correctly placed •² the other two correctly placed $\frac{6}{2} \text{ and } \frac{-\sqrt{36}}{3} \text{ AG (in red so ignore these in marking)}$	The correct places are $ \begin{array}{c c} \hline  & \sqrt{9} \\ \hline  & \frac{12}{3} \\ \hline  & \frac{6}{2} \end{array} $	2
b	•¹ two correctly placed •² another two correctly placed •³ another two correctly placed	The correct places are	3
С	• 0'00 <b>OR</b> 0 00'		1

Ques	stion	Answers	Notes	Total
2	а	(translated horizontally) 3 units to the right (translated vertically) 2 units upwards		
		•¹ both translations mentioned without values or with incorrect values <b>OR</b> both values correct with incorrect transformations	•¹ Correct descriptions like: Horizontal and vertical shift or movement.  ACCEPT down and left. ACCEPT horizontal enlargement 2 and vertical enlargement 3	3
		<ul> <li>one transformation correctly described (with correct direction and value)</li> </ul>	•2 To the right 3 (units)	
		•3 second transformation correctly described (with correct direction and value)	•3 Upwards 2 (units)	
	b	•1 equating with 11	•1 11 = $(a-3)^2 + 2$ . ACCEPT 11 = $(x-3)^2 + 2$	
		•2 one correct step to solve for a	•2 $\pm 3 = (a-3)$ . ACCEPT $\pm 3 = (x-3)$ OR $x(x-6) = 0$	
		•³ their two values for a correct	•3 (a =) 6 and (or) 0 their equation needs to have two values for a to award <b>ECF</b>	
		•4 selecting their correct value of a within the domain	•4 (Selects) <i>a</i> = 6	
			6 without working award 3 marks 6 with one correct step <b>or</b> trial an error statement award 4 marks	4
			If they solve incorrectly and obtain two values for <i>a</i> outside domain then decide to take none of them award • ⁴	
			If they solve incorrectly and their solution allows only one value for a do not award • ⁴ If they solve incorrectly and obtain two values for a and select the one within the domain award • ⁴	

Question	Answers	Notes	Total
3 a	•¹ two correct values <b>OE</b> •² the other two correct values <b>OE</b>	Insert image of the tree diagram stimulus  First ball  Second ball  even  d  odd  d  odd  odd  odd	2
b	•¹ seeing their correct probabilities from (a) •² their result after multiplication of probabilities correct	<ul> <li>1 3/7 and their 2/6</li> <li>2 1/7 OE</li> <li>1/7 OE without working award 2 marks their answer must be less than 1 to award ECF</li> </ul>	2
С	•¹ adding multiplied probabilities to their result in part (b) •² their calculation correct	•1 their $\frac{1}{7} + \frac{4}{7} \times$ their $\frac{3}{6}$ •2 $\frac{3}{7}$ <b>OE</b> (ACCEPT $\frac{18}{42}$ <b>OR</b> 0.42857 <b>OR</b> 43%)  3/7 <b>OE</b> without working award 2 marks <b>SC</b> calculating only odd-odd correctly $(\frac{4}{7} \times \text{their } \frac{3}{6} = \frac{2}{7})$ <b>OE</b> award 1 mark	2

d	$\frac{3+b}{a} = 5 \text{ or } 3+b = 10$	Accept (3+7)/2=5	
		Do not accept 3+7/2=5	1
	AG 7		
е	•¹ setting equation for range <b>OR</b> showing understanding of	•1 $c - a = 34$ <b>OR</b> difference between their a and c is 34	
	concept of range		
	•² setting equation for mean <b>OR</b> showing understanding of	$\bullet^2 \frac{a-12-9+0+3+7+7+10+15+c}{= 2.7}$	
	concept of mean	10	
	•³ simplifying correctly equation for mean	• 3 a + c = 6	
	•4 attempt to solve using substitution <b>OR</b> elimination	•4 Attempt to solve simultaneously, adding <b>OR</b> substitution <b>OR</b> 2c = 40	
		<b>OR</b> $2a = -28$	6
	•5 their value for a or c correct	•5 c = 20	
	•6 the other value for a or c correct	•6 $a = -14$ ALLOW ECF from one of their equations	
		(a =) -14 without working award 2 marks	
		(c =) 20 without working award 2 marks	
		(a =) −14 with one correct step award 3 marks (c =) 20 with one correct step award 3 marks	

Questio	n Answers	Notes	Total
4 a	<ul> <li>•¹ using Pythagoras correctly</li> <li>•² correct calculation         693 AG         Alternative method</li> <li>•¹ using sine rule correctly</li> <li>•² correct calculation         693 AG         Alternative method</li> <li>•¹ recognizing that ∠ABC = 30° and using tan ratio</li> <li>•² correct calculation         693 AG</li> </ul>	•¹ $800^2 = 400^2 + x^2$ <b>OR</b> $800^2 - 400^2$ •² $692.8203$ <b>OR</b> $400\sqrt{3}$ <b>OR</b> $\sqrt{480000}$ <b>OE</b> Do not award •² unless their result can be correctly rounded to $693$ Alternative method •¹ $\frac{x}{\sin 120} = \frac{400}{\sin 30}$ •² $692.8203$ <b>OR</b> $400\sqrt{3}$ <b>OR</b> $\sqrt{480000}$ <b>OE</b> Alternative method •¹ $\tan 30 = \frac{400}{x}$ •² $692.8203$ <b>OR</b> $400\sqrt{3}$ <b>OR</b> $\sqrt{480000}$ <b>OE SC</b> Substituting x with $693$ in Pythagoras <b>OR</b> Sine rule <b>OR</b> tan ratio then making correct calculations both sides award 1 mark	2
b	<ul> <li>•¹ recognizing that triangle OAC is equilateral</li> <li>•OR substituting 400 correctly into circumference formula</li> <li>•² correct substitution in Arc length formula</li> <li>•³ their correct calculation</li> </ul>	•¹ ACCEPT seeing angle 30°, 60° or 120 <b>OR</b> $2\pi400$ <b>OR</b> $800\pi$ •² $\frac{\text{their}120}{360} \times 800\pi$ <b>OR</b> $\frac{120}{360} \times \text{their}800\pi$ (Accept (800/3) x3.14 <b>OR</b> (800/3) x22/7 <b>OE</b> ) •³ 837.758 (838 cm) <b>OR</b> $800\pi/3$ •³ award as <b>ECF</b> only if •¹ <b>OR</b> •² is awarded and the calculation is of similar difficulty	3

			837.758 (838 cm) <b>OR</b> 800π/3 without working award 2 marks	
	С	adding 693 to their value in part (b)	693 + their 837.758 (838 cm) <b>OR</b> 1531 (cm)	
			ACCEPT 692.820 OR 400√3 OR √48000 +their 837.758	1
Que	estion	Answers	Notes	Total
5		x = 5 given in the question see 1 on diagram		
		• 1 2 $x$ + $y$ = 12 one intercept correct <b>OR</b> the line passes through two correct points	y axis	
		•2 $2x + y = 12$ y-intercept correct <b>and</b> crossing the line $x = 5$ at $(5,2)$	11 10 9	
		•3 $2y = x + 12$ y-intercept correct <b>OR</b> the line passes through two correct points	8 7 6 Region 5	
		•4 $2y = x + 12$ y-intercept correct <b>and</b> crossing the line $x = 5$ at $(5,8.5)$		5
		•5 Their correct closed region (Most of the icon "region" has to be in the correct region in order to award the mark)	•5 <b>ECF</b> can be awarded for a region to the left of $x = 5$ and above their $2x + y = 12$ and below their $2y = x + 12$	

Que	estion	Answers	Notes				
6	а	•¹ calculation of tax for 30 % band	Annual income bands in EUR	Tax rate	Calculation of tax	Amount of tax EUR	
		calculation of tax for 60 70 barra	0 < income ≤ 6000	0 %	0	0	
	●2 their amount of tax for 30 %		6000 < income ≤ 12 000	5.5 %	(12 000 – 6000) x 5.5 %	330	
			12 000 < income ≤ 25 000	14 %	(25 000 – 12 000) x 14 %	1820	
	band calculated correctly		25 000 < income ≤ 70 000	30 %	(70000-25000)x30%	13500	
	band calculated correctly		Above 70 000	41 %	(80000-70000)x41%	4100	
		•3 calculation of tax for 41 % band			Total tax paid on 80 000 EUR	19750 ·	
		•4 their amount of tax for 41 % band calculated correctly from a subtraction followed by multiplication •5 their amount of total tax paid calculated correctly					5

Questi	ion	An	swers	Notes		Total
6 b		Identify Factors (F)  Calculation Home (H) For scenario 1  Calculation Relocate (R) For scenario 2	Attempt to calculate Example: Calculate Total Tax Tax: 330 + 1820 + OR Calculate Total exp (400 + 900 + 500)x OR 40000-their calculate Example: Expenses: (500+900+80+600) OR Calculate correctly (500+900+80+600) OR Assuming a certain error in calculation (500+900+80+600) Accept 2080+150= OR	enses, the relocation place <b>OR</b> family  e monthly <b>OR</b> yearly net saved value  c correctly: 15000 x 0.3(= 6650)  enses correctly 12(=21600)  ted taxes-their calculated yearly expenses e monthly <b>OR</b> yearly net saved value  *12+their2x150=24960+300=25260  total expenses excluding home travel *12(=24960)  number of times to visit home but making an eg. +their2x150=2080+300=2380	Identify more than two elements from: Income, taxes, expenses, the relocation place OR family	10
		Justify accuracy of calculations (J)	Weak unsupported OR Sensible rounding OR Recognizing that taccurate	•	Recognizing the calculation is not accurate with good reasoning: because many of the monthly expenses are estimates because medical expenses can vary because the number of journeys home vary	

Decision (D)	Relocate <b>OR</b> do not relocate with weak supporting justification	Relocate or do not relocate with good supporting
	Examples:	justification (balanced reflection)
	I will relocate because I will make more money	
	I will not relocate because I don't want to move	Examples:
		I will relocate because I will make more money even if I
	Relocate <b>or</b> do not relocate without justification <b>OR</b> with irrelevant	come to visit home once a month
	justification award 0 marks	
		I will not relocate because even though I will be making
		xxx more money, it is not much compared to staying with
		family.
		I will relocate because I will make more money and I will
		gain new experiences by moving

estion	Ans	swers	Notes		Total
а	<ul> <li>any correct area</li> <li>the corresponding other correct area</li> <li>show appropriate operation either adding OR subtracting</li> <li>AG</li> </ul>		SC seeing only:  1800 – 1000 = 800 award 2 marks  OR  600 + 200 = 800 award 2 marks  60x10+30x8-40=800 award 1 mark for 60x10		3
	<ul> <li>4 x 10 x 25 = (1000)</li> <li>(30 x 60) - 4 x 10 x 25</li> <li>Total 800 AG</li> </ul>	<ul> <li>60 x 10 (= 600)</li> <li>10 x 10 x 2(= 200)</li> <li>60 x 10 + 10 x 10 x 2</li> <li>Total 800 AG</li> </ul>	` ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '	<ul> <li>10 x 10 x 3 (= 300)</li> <li>10 x 25 x 2 (= 500)</li> <li>300 + 500</li> </ul>	
b	$\tan\theta = \frac{30}{60}$		Accept $(\tan \theta =)\frac{15}{30}$		1
С	<ul> <li>•¹ using tan ratio to determine correct value of θ</li> <li>•² using correct ratio to set equation in x</li> <li>•³ rearranging x on one side correctly</li> <li>•⁴ the correct value of x</li> <li>•⁵ rounding their result correctly to 1 dp</li> </ul>		•1 $(\arctan \frac{1}{2} =) 26.565 \deg$ •2 $\sin(\text{their}\theta) = \frac{3}{x} \ \text{OR} \ \frac{3}{\sin(\text{their}\theta)} = \frac{x}{\sin 90}$ •3 $x = \frac{3}{\sin(\text{their}\theta)} \ \text{OR} \ x = \frac{3}{\text{their}0.447}$ •4 $(x =) \ 3\sqrt{5} \ \text{OR} \ 6.708 \ (cm)$ •5 $(x =) \ 6.7 \ (cm) \ 1 \ dp$ $(x =) \ 3\sqrt{5} \ \text{OR} \ 6.708 \ (cm) \ without \ works$		5

	Alternative method	Alternative method	
	$ullet^1$ using Pythagoras to determine correct value of $\sin \theta$	• $\sin \theta = \frac{1}{\sqrt{5}}$ <b>OR</b> 0.447	
	•² using correct ratio to set equation in x	•2 $\frac{3}{x}$ = their $\frac{1}{\sqrt{5}}$ <b>OR</b> $\frac{3}{\text{their sin }\theta} = \frac{x}{\sin 90}$ <b>OR</b> $\frac{3}{x}$ = their 0.447	
	•³ rearranging <i>x</i> on one side correctly	•3 $x = \frac{3}{\text{their } \sin \theta}$ <b>OR</b> $x = \frac{3}{\text{their } 0.447}$	
	• ⁴ the correct value of <i>x</i>	•4 $(x =) 3\sqrt{5}$ <b>OR</b> 6.708 (cm) •5 $(x =) 6.7$ (cm) 1 dp	
	•5 rounding their result correctly to 1 dp	Seeing only $\frac{3}{\sin 26.565}$ = 6.708 award 4 marks	
		Seeing only $\frac{3}{\sinh \theta}$ = their value not rounded <b>OR</b> exact value that	
		doesn't need rounding award 3 marks	
d	•1 calculating their base of the triangle correctly	•¹ (Base of triangle = 25-their 6.7 =) 18.3	
	•² using trigonometric ratio correctly to set equation involving H		
		$\frac{H}{\sin(\text{their } 26.565)} = \frac{\text{their } 18.3}{\sin(90 - \text{their } 26.565)}$	
	•³ rearranging their equation correctly to have H on one side	•3 $(H =)\frac{1}{2} \times \text{their } 18.3 \text{ OR } \frac{25 - 3\sqrt{5}}{2} \text{ OR } 9.146$	4
		•² has to be seen in order to award •³.	
	• alculating correctly the area of triangle using their height of triangle and their base of triangle	To award ●⁴ their H without working has to be in the range of [9.1 , 9.2]	

	84 AG	•⁴ $(T_1 =)\frac{1}{2} \times$ their 18.3× their 9.146 = 83.6 Accept 83.7  Do not award •⁴ unless their result can be correctly rounded to 84  Do not award •⁴ if their answer is exactly 84	
е	•¹ seeing 4 x 44 <b>and</b> 4 x 84 •² Adding correctly their multiplied values	•² 512 (cm²) 512 without working award 2 marks	2
f	•¹ dividing by total area of flag •² calculating their percentage correctly	<ul> <li>•¹ Seeing 60 x 30 <b>OR</b> 1800 in denominator</li> <li>•² (% represented = their 512/their 1800 x 100=)28.444(%)</li> <li>28.444 <b>OR</b> 28 % without working award 2 marks</li> </ul>	2

Que	stion	Answers	Notes	Total
8	a	•¹ one suitable pattern •² another suitable pattern	Examples of suitable patterns:  Differences are 2,4,6  All even numbers  Difference increasing  Difference is even  Second difference constant  Number of female times the previous number of female f multiplied by f-1  SC 2 marks: Second difference is 2  The differences are two step different OE  Double of triangular numbers  Do not accept Number of kisses increases as the number of females increases OE	2
	b	•¹ correct general rule with poor notation •² correct general rule with correct notation	<ul> <li>•¹ (k =)f*(f-1) OR f²-f OR ff-f OR f*f-f</li> <li>•² (k =) f(f-1) OR (k=)f²-f OR fxf-f OR fx(f-1) OR (f)(f-1)</li> <li>Do not accept description in words</li> <li>Do not accept notation that reads incorrect general rule eg. f x f-1</li> </ul>	2
	С	<ul> <li>•¹ substitute f≥ 6 into their rule</li> <li>•² correctly calculate their value of k after substitution f≥ 6</li> <li>•³ recognise that their result is the same as their predicted value</li> </ul>	•³ ACCEPT seeing the 30 in the table and seeing k = 30 from their calculation <b>SC</b> if "tested" correctly award 1 mark Tested correctly is when they apply the steps of verification mentioned in the left column on a value of $f \le 6$	3

8	d	M ar k	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/proof (J)	Notation and terminology (N)	Communic ation (C)	
		1	Correctly predict one term for any two sets of data <i>s</i> or <i>b</i> or <i>k</i> or <i>G</i> (when <i>m</i> ≥ 6)	Attempt to describe a pattern Ex: they are all increasing or Recognize that pattern for s and b are the same	Attempted to test their described pattern or general rule of $b$ or $s$ or $G$ for $m \le 5$ ex: substitutes in their rule value of $m \le 5$	Attempt to verify their general rule of $b$ or $s$ or $G$ for $m \ge 6$ ex: substitute in their rule value of $m \ge 6$	Attempt to justify any of their described patterns or general rules Ex: refer to difference between terms or test at least two values for s and say it works or after stating the rule they say every time it is male times male	The notation or terminology is correct  OR  The notation and terminology have significant errors	Lines of reasoning are incomplete OR incoherent.  Ex: Only two lines of calculations or algebraic steps	
		2	Correctly predict one row for the four sets of data (when $m \ge 6$ )  OR  Correctly predict four values in the columns of $s$ or $b$ or $k$ or $G$ (when $m \ge 6$ )	Recognize that pattern for <i>s</i> and <i>b</i> are the same <b>and</b> Attempt to describe a pattern for <i>s</i> Ex: the number is multiplied by itself or the number increases in 3,5,7,	Correctly calculating their value of <i>b</i> or <i>s</i> or <i>G</i> using their value of <i>m</i> ≤ 5 and recognizing that their result is the same as the table value (ACCEPT seeing their calculated value and their predicted value in the table being equal)	Correctly calculate their value of <i>b</i> or <i>s</i> or <i>G</i> using their value of <i>m</i> ≥ 6	Justify any of the general rules correctly Ex: The second difference is constant Ex: The s (or b) are the square numbers OR Attempt to justify the general rule for Greeting (G) Ex: Attempt to add any of their rules for b, s or k together or test at least two values for G and say it works	The notation of a general rule and terminology describing pattern is correct  OR  Correct general rule in correct notation but not simplified Ex:  G=m(2m+m-1)	Lines of reasoning are coherent OR answer is organised using a logical structure.	24

M ar k	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/proof (J)	Notation and terminology (N)	Communic ation (C)
3	Correctly predict two rows for the four sets of data (when $m \ge 6$ )  OR  Correctly predict eight values in the columns of $s$ or $b$ or $k$ or $G$ (when $m \ge 6$ )	Correctly describe pattern in words for smiles (s)  Ex: They are the square numbers or the second difference is constant or it is quadratic or second degree or $s = m \times f$		Recognise that their result for <i>G</i> is the same as their predicted value (ACCEPT seeing their calculated value and their predicted value in the table being equal)  Allow V3 only if they verify their rule for <i>G</i> correctly	Justify any of the general rules correctly Ex: The second difference is constant Ex: The s (or b) are the square numbers AND Attempt to justify the general rule for Greeting (G) Ex: Attempt to add any of the rules for b, s or k together or test at least two values for s and say it works OR Justify correctly the general rule for Greeting (G) Ex: Add correctly their rules for b, s and k together	The notation of their general rule for $G$ and terminology used are correct and their general rule is simplified as $G = 3m^2 - m$ or $G = m(3m-1)$ Award only if D5 is achieved	Lines of reasoning are coherent AND answer is organised using a logical structure  Award only if J2 is achieved
4		Correctly describe pattern as general rule for smiles ( $s$ ) <b>AND</b> recognise it is the same as for bows ( $b$ ) $s = m^2$ and $b$ the same writing only $s = m^2$ award D3			Justify any of the general rules correctly Ex: The second difference is constant Ex: The s (or b) are the square numbers <b>AND</b> Justify correctly the general rule for Greeting (G) Ex: Add correctly the rules for b, s and k together		

M ar k	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/proof (J)	Notation and terminology (N)	Communic ation (C)
5		Attempt to describe pattern as general rule for <i>G</i> Ex: the rule is $3m^2$ - $m$ or $m(3m$ - $1)$ OR adding correct rules incorrectly $G=2m^2$ - $m+m^2=2m^2$					
6		Correctly describe the pattern for $G$ as a general rule $G = m(m-1) + m^2 + m^2$ <b>OR</b> $G = 3m^2 - m$					

# Predictions

Number of males (m)	Number of females	Number of kisses (k)	Number of bows (b)	Number of smiles	Total number of greetings (G)
1	1	0	1	1	2
2	2	2	4	4	10
3	3	6	9	9	24
4	4	12	16	16	44
5	5	20	25	25	70
6	6	30	36	36	102
7	7	42	49	49	140
8	8	56	64	64	184
9	9	70	81	81	232
10	10	86	100	100	286
11	11	110	121	121	352
12	12	132	144	144	420
13	13	156	169	169	494
14	14	182	196	196	574
15	15	210	225	225	660

# Rules:

$$k = m(m-1) \qquad \qquad s = m^2$$

$$b = m^2$$

$$G = m(m-1) + m^2 + m^2 = 3 m^2 - m$$